## ENSC 201 Quantitative Analysis Exercise Answers

\{When correcting, use the mark scores if you wish. Total: 100 marks with 5 marks for proper assignment formatting\}

1. Significant Figures: Round to 3 significant figures: $\{3$ total -1 each $\}$
(a) $1397=1.40 \times 10^{3}$

Using scientific notation is the only correct way to answer this question as the number of significant figures is now clear. As there are no restrictions on how to express your result so you can have the same answer for questions 1(a) and 2(a).
(b) $1.6805=1.68$
(c) $.00013340=0.000133$
2. Scientific Notation: Write your answers to Question 1 in scientific notation: $\{3$ total -1 each $\}$
(a) $1.40 \times 10^{3}$
(b) $1.68 \times 10^{0}$
(c) $1.33 \times 10^{-4}$
3. Calculations and Calculator Use: Solve as indicated and then express your final answer in scientific notation. [The rules indicated by number below and in the Lab 0 "Sig Fig" link refer to Appendix B, see "B. 8 Significant Figures". For all calculations: Don't' round interim results. During calculations retain more significant figures than needed (calculators carry as many as they have memory for). Only round for the correct number of significant figures at the end. See the example below for best practices: Report your answer with too many significant figures and then report it after rounding for significant figures] \{8 total - 2 each $\}$
(a) $\frac{4.71 \times 10^{3}}{3.52 \times 10^{2}}=13.3807 \rightarrow$ sig. figs. $=13.4$ or $1.34 \times 10^{1}$ [rule 5]
(b) $56.3 \times 78.7=4,430.81 \rightarrow$ sig. figs. $=4,430$ or $4.43 \times 10^{3}$ [rule 5]
(c) $100 \cos 20^{\circ}=93.969 \rightarrow$ sig. figs. $=90$ or $9 \times 10^{1}$ [rule 5]
(d) $\frac{500^{\frac{1}{4}}}{5.67 \times 10^{-8}}=83,398,730.95 \rightarrow$ sig. figs. $=80,000,000$ or $8 \times 10^{7}$ [rule 5]
4. Temperature Conversions: Refer to Appendix C. Show your formulas and workings for full marks. Note how the decimal is indicated in each question. Report your answer as the end result of your calculation and then report it again adjusted for significant figures. $\{6$ total -2 each yellow answer\}
These answers are rounded to their correct number of significant figures (sig. fig.) at the end. As done for question 3, best practices report both the calculation result with too many significant digits and its significant figure result. Reasons to do this: 1) calculation errors are obvious. 2) Final answers for one question often become interim results for another. See Appendix B for information on determining significant figures; a summary of these rules is also posted on the website.

Convert to Kelvin (K): Formula: $\mathrm{K}=$ value in ${ }^{0}$ Celsius +273.15 or $\mathrm{K}={ }^{0} \mathrm{C}+273.15$
(a) $-25.00{ }^{\circ} \mathrm{C}$
$-25.00^{\circ} \mathrm{C}=(-25.00+273.15)=248.15 \mathrm{~K}=248.15 \mathrm{~K}$
[Sig. Fig. rule 2 and 4] Explanation: Rule 2, the constant 273.15 K , is an exact not a measured value, so it never limits the number of significant digits. According to rules 2 and 4 , the measured value $\left(-25.00^{\circ} \mathrm{C}\right.$ ) limits the number of significant figures to 2 decimals as an answer cannot become more accurate by adding a constant. Only measured values influence the number of significant figures in a final result.

Note that the size of a Kelvin is the same as the size of a Celsius degree; 273.15 is the difference between the two systems.

Convert to Celsius: Formula: ${ }^{\circ} \mathrm{C}=$ value in Kelvin -273.15 or ${ }^{\circ} \mathrm{C}=\mathrm{K}-273.15$
(b) $200 . \mathrm{K} \quad 200 . K=(200 .-273.15)=-73.15{ }^{\circ} \mathrm{C}=-73 .{ }^{\circ} \mathrm{C}$
[Sig. Fig. rule 2 and 4] Explanation: Similar to (a) above. The final answer has no digits after the decimal because a measured value cannot gain precision (more decimals) by adding or subtracting (rule 4). In this case the measured value of 200 . K limits the number of digits after the decimal to none.
Note: If we were multiplying /dividing, 200.K has 3 sig figs while 200 K only has 1 significant figure. Also, reporting a measurement with a decimal where all the digits are to the left of the decimal is now considered uncommon. Scientific notation ( $2.00 \times 10^{2}$ ) represents this measurement more clearly.)

Convert to Fahrenheit: ${ }^{\circ} \mathrm{F}=\left(\frac{9}{5}\left(\right.\right.$ Temp in $\left.\left.{ }^{\circ} \mathrm{C}\right)\right)+32^{\circ}$
[Note: The values $9 / 5$ and 32 are exact constants and don't limit the number of significant figures in a final answer so rule 2 applies. If you don't know which constants are exact - look them up.]

$$
\text { Convert Kelvinto Celsius } \quad 261.0 \mathrm{~K}=(261.0-273.15)=-12.15^{\circ} \mathrm{C}
$$

(c) 261.0 K

Convert Celsiusto Fahrenheit $\quad{ }^{\circ} \mathrm{F}=\frac{9}{5}\left(-12.15{ }^{\circ} \mathrm{C}\right)+32^{\circ}=10.13^{\circ} \mathrm{F}=10.1^{\circ} \mathrm{F}$
(2)
[Sig. Fig. rule 4, 2, and 5] Explanation: Significant figures in this multi-step calculation are not considered until the final answer. The only measured value is 261.0 K . As 261.0 K has 1 decimal, it limits the end result significant figures to 1 decimal when adding and subtracting (rule 4). Rule 2 indicates that none of the constants affect the end result significant figures. The least precise measured value ( 261.0 K ) has 4 significant digits so rule 5 (multiplying and dividing) has a lower priority than rule 4 (addition and subtraction).
5. Dimensional Analysis: Refer to Appendices B and C for assistance. Complete the blanks by expressing the following combinations of dimensions in S.I units and answer the questions. Make sure to understand the difference between basic and derived units (see lists in Appendix C). \{10 total - 1 each non italic answer\}
(a) Indicate the basic SI units for MLT-2? $\mathrm{kg} \mathrm{m} \mathrm{s}^{-2}$

What is something with these dimensions called? Force (... in Newton units)
(b) Indicate the basic SI units for $\mathrm{L}^{2} \mathrm{~T}^{-1}$ ? $\quad \mathrm{m}^{2} \mathrm{~s}^{-1}$

What S.I. units can multiply your answer and result in velocity? $\mathrm{m}^{-1}$ (The units of velocity are $\mathrm{m}^{-1}$; so to get velocity you need to multiply by $\mathrm{m}^{-1}$ )
(c) Indicate the most derived SI units for the dimensions $\mathrm{QL}^{-2} \mathrm{~T}^{-1} . \mathrm{W} \mathrm{m}^{-2}$
(The most basic form of this entity would be $\mathrm{MT}^{-3}$ (from $\mathrm{ML}^{2} \mathrm{~T}^{-2} \mathrm{~L}^{2} \mathrm{~T}^{-1}=\mathrm{MT}^{-3}$ ))
What is something with these dimensions called? heat flux density
(the amount of energy per unit time entering a unit area).
Use other S.I. units and express $\mathrm{Q} \mathrm{L}^{-2} \mathrm{~T}^{-1}$ in another form. $\mathrm{Js}^{-1} \mathrm{~m}^{-2}$ or $\mathrm{kg} \mathrm{s}^{-3}$
(d) Indicate the SI units for the dimensions $\mathrm{Q} \theta^{-1} \mathrm{~L}^{-3}$ in derived form $\mathrm{J} \mathrm{K}^{-1} \mathrm{~m}^{-3}$
[Note: a Joule $=\mathrm{kg} \mathrm{m}^{2} \mathrm{~s}^{-2}$ so the answer $\mathrm{kg} \mathrm{m}^{-1} \mathrm{~s}^{-2} \mathrm{~K}^{-1}$ is the most basic form of the answer]
Which S.I. units can divide these units to give dimensions of specific heat (which is $Q M^{-1} \theta^{-1}$ )? $\mathrm{kg} \mathrm{m}^{-3}$

What is this quantity called? density
6. Conversions: Report units and conversion factors. Clearly show your work by reporting conversions used and intermediary results. \{12 total -2 marks each $\}$

Determine the:
(a) number of seconds in 1 year $3.1557600 \times 10^{7} \mathrm{~s}$ (for this question use the most accurate length of a year: 1 year $=365.25$ days)

$$
\left(\frac{365.25 \text { days }}{1 \text { year }}\right)\left(\frac{24 \text { hours }}{1 \text { day }}\right)\left(\frac{60 \mathrm{~min}}{1 \text { hour }}\right)\left(\frac{60 s}{1 \mathrm{~min}}\right)=3.1557600 \times 10^{7} s
$$

(b) the number of square meters in 3.2 square kilometers $3.2 \times 10^{6} \mathrm{~m}^{2}$

$$
3.2 \mathrm{~km}^{2} \times\left(\frac{1000 \mathrm{~m}}{\mathrm{~km}}\right)^{2}=3,200,000 \mathrm{~m}^{2}=3.2 \times 10^{6} \mathrm{~m}^{2}
$$

(c) pressure in hPa of 28.77 inches of mercury $\quad 974.3 \mathrm{hPa}$

Use the conversions: 1 atmosphere $=1013.2 \mathrm{hPa}=29.92$ " Hg

$$
=(28.77 \text { in } \mathrm{Hg})\left(\frac{1013.25 \mathrm{hPa}}{29.92 \text { in } \mathrm{Hg}}\right)=974.3049 \mathrm{hPa}=974.3 \mathrm{hPa}
$$

(d) number of quacktics in a goobric $\quad 1.59 \times 10^{-3}$ quacktics

Where: 1 quacktic $=3.45$ lantex 1 motoxic $=100.00$ soomacs 1 lantex $=0.456$ motoxics And by definition: 1 soomac $=4$ goobrics

$$
\begin{aligned}
& 1 \text { goobric }\left(\frac{1 \text { soomac }}{4 \text { goobrics }}\right)\left(\frac{1 \text { motoxic }}{100.00 \text { soomacs }}\right)\left(\frac{1 \text { lantex }}{0.456 \text { motoxics }}\right)\left(\frac{1 \text { quacktic }}{3.45 \text { lantex }}\right)=.00158912 \text { quacktics } \\
& =1.58912 \times 10^{-3} \text { quacktics }=1.59 \times 10^{-3} \text { quacktics }
\end{aligned}
$$

(e) subtract: $27^{0} 36^{\prime} 22^{\prime \prime}-3^{0} 46^{\prime} 52^{\prime \prime}$
(Remember: $I^{0}=60$ ' $l^{\prime}=60$ " and: ${ }^{0}$ means degrees; ' means minutes; " means seconds)

|  |  | $95^{\prime}$ | $82^{\prime \prime}$ |
| :--- | :--- | :--- | :--- |
| best rewritten as $\rightarrow$ | $26^{\circ}$ | $35^{\prime}+60$ | $22+60$ |
|  | $27^{0}$ | $36^{\prime}$ | $22^{\prime \prime}$ |
|  | $-3^{0}$ | $46^{\prime}$ | $52^{\prime \prime}$ |
| $23^{\circ}$ | $49^{\prime}$ | $30^{\prime \prime}$ |  |

(f) report the degrees, minutes, and seconds of latitude in 53.77 degrees latitude (Remember: $1^{0}=60{ }^{\prime} I^{\prime}=60$ " and: ${ }^{0}$ means degrees; ' means minutes; " means seconds)

$$
53.77^{0} \rightarrow 53^{0}+\left(0.77^{\circ}\right)\left(\frac{60^{\prime}}{1^{\circ}}\right)=46.2^{\prime} \rightarrow 46^{\prime}+\left(0.2^{\prime}\right)\left(\frac{60^{\prime \prime}}{1^{\prime}}\right)=12^{\prime \prime}
$$

Putting it together: $53.77^{0}=53^{\circ} \quad 46^{\prime} 12^{\prime \prime}$
7. Time Conversions / Calculations: Show/explain how you got your answers. $\{8$ total -4 each $\}$ Coordinated Universal Time (UTC), formerly known as Greenwich Mean Time (GMT) is used in meteorology to avoid confusing different local time zones when looking at weather observations from around the globe. In our part of the world, Pacific Standard Time (PST) is 8 hours behind UTC, and Pacific Daylight Time (PDT) is 7 hours behind UTC. The exact date for changing from between Standard Time and Daylight Savings time varies yearly but generally in the Pacific time zone we change to Daylight Savings Time (i.e. "spring forward") around mid-March, and change to Standard Time (i.e. "fall back") during the second week in November.

What is the local time and date for April 30 at 06:30 UTC?
Daylight Savings Time is 7 hours behind UTC; so the local time is April 29 23:30 PDT
What is the UTC time and date of January 20, 17:10 PST?
UTC is 8 hours ahead of PST; so the UTC time is January 21 at $01: 10$ UTC
8. Graphing \{11 marks graph, 3 marks interpretation -- 14\}:

Air temperatures were measured over two days at the 6 am ( 0600 hours), noon ( 1200 hours), and 5 pm (1700 hours); these data form a two-day temperature time series. Correctly graph the data on $10 \mathrm{~mm} \times 10 \mathrm{~mm}$ graph paper (provided in the lab). Use a scale of $1 \mathrm{~cm}=1$ degree Celsius $\left({ }^{0} \mathrm{C}\right.$ ) for the temperature axis ( Y -axis), and $1 \mathrm{~cm}=2$ hours (hr) for the time axis ( X -axis). Follow appropriate graphing conventions. Give graphs a descriptive title, proper axis divisions and labels (names, units and their abbreviations), plot points properly, and provide a legend as needed.

| $\mathrm{Y}\left({ }^{\circ} \mathrm{C}\right)$ | -10 | -3 | -6 | -5 | 3 | -2 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| X (time as hr) | Jan 3, | Jan 3, | Jan 3, | Jan 4, | Jan 4, | Jan 4, |
|  | 0600 | 1200 | 1700 | 0600 | 1200 | 1700 |

Complete the plot by connecting the points to show the time series. Then, plot the average temperature for each day and for the entire two day time-period. Clearly distinguish the graphed lines using appropriate graphing conventions. What changes in weather conditions might explain the differences observed in the recorded temperatures?

For those using marks to evaluate: Question 8 mark breakdown (10 marks total)
3 -- plot ( 0.5 deducted for each error)
3 - axis labels (1.5 each axis: axis name, axis divided into units, axis unites are stated)
1 - title - should be as descriptive as possible
1.5 - daily average termperatures $\times 2$ + average temperature over the 2 days ( 0.5 each)
0.5 -.legend or labels to distinguish graphed average value lines

1 - explanation of weather conditions that account for differences between the 2 day's temperatures.


- While there are other ways to graph this data, the question asks you to plot a time series, so this is the correct representation. A time series plot is a specific type of graph that sequentially shows the pattern of observations through time.
- Daily averages are computed by adding each day's 3 measured values and dividing by 3. Similarly, the average over the two-day period adds all 6 measurements and divideds by 6 . On the plot, the two daily averages could be joined by a vertical line but it is equally correct to leave them separated.

Possible changes in weather conditions that might explain the recorded temperature pattern are:

1) Daily heating due to solar radiation patterns.
2) The warming trend from Jan 3 to Jan 4 indicates warmer air moving into the area likely caused by frontal activity. More complicated changes in cloud cover could also create this pattern (cooler nights and warmer days are generally clearer).
9. Plotting, Mapping, and Contouring (34 marks):

The following temperature measurements are taken at different stations at the same time. Plot these on the provided graph paper and contour the resulting temperature pattern in $1^{\circ} \mathrm{C}$ intervals.

## Steps:

- Create a base map with a scale of .01 degree longitude $=1 \mathrm{~cm}$; and .01 degree latitude $=2 \mathrm{~cm}$. This is where you will plot the temperature values given below. [Note the scale given here is an approximation of the real world scale. To create a correct scale you would use .01 degrees $=1.7$ cm . We won't do this as it is much more awkward to graph without computer graphing programs or specially adjusted paper.] Remember both graph axes should be oriented correctly for their hemispheres, i.e. latitude degrees increase toward the north, and longitude degrees increase toward the west. Properly label and title your map.
- Locate each station on your base map with a dot. Write its name and the accompanying temperature observation beside each dot.
- Contour the temperature pattern by drawing smooth temperature contours (isotherms) on the map to show the spatial distribution of temperature.

| Station | Longitude | Latitude | Temperature |
| :--- | :--- | :--- | :--- |
| BC Rail | 122.74 W | 53.883 N | -12.5 C |
| CBC | 122.70 W | 53.905 N | -9.2 C |
| Gladstone | 122.76 W | 53.860 N | -8.2 C |
| Jail | 122.71 W | 53.905 N | -10.1 C |
| Lakewood | 122.80 W | 53.917 N | -12.9 C |
| Plaza400 | 122.74 W | 53.913 N | -13.2 C |
| UNBC | 122.82 W | 53.888 N | -7.5 C |

## For those using marks to grade:

Question 9 mark breakdown ( 34 marks total)

> 4 - longitude values are graphed so they are spatialy correct.
> 2 - required scales are used for graphing
> 1 - title - should be as descriptive as possible
> 3 -- axis labels (1.5 each axis: axis name, axis divided into units and West / East indicated, axis unites are stated)
> 14 - plotting correctly done, points are named, in the correct locations, and temperatures are indicated correctly ( 2 marks / point)
> 8 - contouring well done (1 mark per line)
> 2 - correct contour interval used (contoured at $1^{0}$ intervals \& labels)


1. To represent longitude values realistically, they must decrease as you move away from the graph's x-axis origin (the opposite of most graphed data where zero starts at the left and increases as you move right).

## ENSC 201 LAB 1: Radiation Relationships - Questions 10 \& 11 ANSWERS

This answer key contains additional explanations that enhance understanding of either the lab content or the learning process. Required answer elements are highlighted in yellow. In question 11, where answers differ by lab section, answers are colour coded by lab days and start times. Representing mathematical processes with words is acceptable when you are not sure how to properly express mathematical steps, however, before exams you should visit your instructor for help with determining how to clearly report your mathematical work.

## ANSWERS Part A: Solar Radiation Inputs:

10. Question: Use the Cosine Law of Illumination to calculate the flux density of solar radiation arriving at the top of the atmosphere over Prince George (without attenuation) on the afternoon of Sept. 21 at 3:00 p.m. PST (15:00 PST). For simplicity assume it is not a leap year, and that local standard time is the same as solar time. Prince George is located at $122^{\circ} 41^{\prime} \mathrm{W}$ longitude and $53^{\circ} 53^{\prime} \mathrm{N}$ latitude.

In your answer report latitude and longitude in degrees, minutes and seconds. Remember: 1
degree $=60$ minutes $\left(1^{\circ}=60^{\prime}\right)$ and 1 minute $=60$ seconds $\left(1^{\prime}=60^{\prime \prime}\right)$

The question states: Use the Cosine Law of Illumination to determine the solar flux density at the top of the atmosphere over Prince George at 3 pm on Sept. 21. Note:

- it is not a leap year (which would matter for dates after Feb. 28);
- local time = the solar time (so solar noon = local noon; in reality these are close but not exactly the same).

You might also notice that September 21 is the Fall Equinox - one of the two days each year when the Sun is directly overhead at the equator at solar noon. (The term Equinox is derived from equal night because at this time, days and nights are approximately the same length around the world.)

From the Part A) Background information, the Cosine Law of Illumination is:
$I=I_{0} \cos Z$
where:
$I_{0}=1361 \mathrm{Wm}^{-2}=$ the maximum solar flux density (i.e. at the top of the atmosphere)
$I=$ the incident radiation received by the surface at a particular location
$\cos Z=\sin \phi \sin \delta+\cos \phi \cos \delta \cos h$-- this is the cosine of the zenith angle ( $Z$ )
where the components of $\cos Z$ are:
$\phi=$ the latitude of your location, in this case - Prince George. Convert the location (given in degrees \& minutes) to it decimal degrees form.

$$
\emptyset=53^{\circ} 53^{\prime} N=53^{\circ}+\left(53^{\prime}\right)\left(\frac{1^{\circ}}{60^{\prime}}\right) N=53.8833^{\circ} N
$$

$h=$ the hour angle which changes by $15^{\circ}$ per hour from solar noon (in this question we've assumed that solar noon = local noon; solar noon is the time when the Sun is highest in the sky at your location.). Solar noon has an hour angle of zero, meaning that the sun is over the longitude of the solar noon's location. So, use the above information and logic to determine that:
$15: 00$ PST is 3 hours past noon, so $\rightarrow h=\left(+3\right.$ hours $\times 15^{\circ} /$ hour $)=+45^{\circ}$
$\delta=$ the solar declination; this is the latitude (on Earth) where the Sun is directly overhead. A positive value indicates degrees North ( ${ }^{\circ}$ North) and a negative value indicates degrees South ( ${ }^{\circ}$ South). Determine $\delta$ using:

$$
\delta=-23.4^{0} \cos \frac{\left(360\left(T_{J}+10\right)\right)}{365}
$$

$$
\text { where } T_{J}=\text { the Julian Day }
$$

So $T_{J}$ for Sept $21^{\text {st }}$ is: $T_{J}=(31+28+31+30+31+30+31+31+21)=264$

$$
\delta=-23.4^{0} \cos \frac{(360(264+10))}{365}=-0.1007^{\circ}
$$

If you need to determine $\delta$ during a leap year, the equation becomes:

$$
\delta=-23.4^{0} \cos \frac{\left(360\left(T_{J}+10\right)\right)}{366}
$$

$-0.1007^{0}=0.1007^{\circ}$ South (or just South of the equator)
This answer ( $-0.1007^{\circ}$ or $0.1007^{\circ} \mathrm{S}$ ) is reasonable for $\delta$ on Sept $21^{\text {st }}$ because it is the Fall Equinox. At the Equinox we expect the declination to be very close to zero as the Sun is directly over the Equator.
Now use the values $(\phi, h$, and $\delta)$ to determine $\cos Z$ :

$$
\begin{aligned}
& \cos Z=\sin \phi \sin \delta+\cos \phi \cos \delta \cos h \\
& \cos Z=\sin \left(53.8833^{\circ} \mathrm{N}\right) \sin \left(-0.1007^{\circ}\right)+\cos \left(53.8833^{\circ} \mathrm{N}\right) \cos \left(-0.1007^{\circ}\right) \cos \left(45^{\circ}\right) \\
& \cos Z=0.4153
\end{aligned}
$$

Then substitute $\cos Z$ and $I_{0}$ into the Cosine Law of Illumination ( $I=I_{\circ} \cos Z$ ) to determine the solar flux density over Prince George on Sept. 21 at 3 pm :

$$
I=I_{0} \cos Z=\left(1361 \mathrm{Wm}^{-2}\right)(0.4153)=565.22 \mathrm{Wm}^{-2}=565 \mathrm{Wm}^{-2} \text { on Sept } 21 @ 15: 00 \text { PST }
$$

11. Question: Identify the latitude and longitude of the location where the Sun is directly overhead at the start of your lab period. Use Background Information, previous question information, and the fact that Earth's daily revolution is 15 degrees of longitude per hour.
This comes from Earth rotating once per day, or $360^{\circ}$ of longitude per 24 hours, so:

$$
\frac{360^{\circ}}{24 \text { hours }}=\frac{15^{0}}{1 \text { hour }}
$$

(Note: the hint given in the question is omitted here.)
Check your answer to Question 11 by matching the answers below for your lab's day (gives the latitude where the sun is overhead, and your lab's time (gives the longitude where the sun is overhead).

The answer key demonstrates a detailed calculation for the first lab section and less detailed answers for the others; see the first lab section answer for all the steps in the calculation. The process is the same for all lab section answers except the:

- day of your class changes the latitude value, and
- time of your class changes the longitude value.

To determine the latitude and longitude where the Sun is directly overhead, you must understand that parts of the Cosine Law of Illumination indicate where the Sun is directly overhead. Since our classes are in January, leap years don't matter. Also for simplicity, we will assume that solar noon is the same as local noon.

The latitude where the Sun is directly overhead is the solar declination $(\boldsymbol{\delta})$ - revisit the definition of $\delta$. Because the latitude where the Sun is overhead changes with the season, the time of year (or date) determines $\delta$.

For Tues Jan $9^{\text {th }} 2024$ the latitude where the Sun is directly overhead is:

$$
\begin{aligned}
& T_{J}=9 \\
& \delta=-23.4^{0} \cos \left(360\left(T_{J}+10\right)\right) \\
& 365 \\
& \text { [For those wondering about significant figure determinations: The } \\
& \text { values } 10,360,365 \text {, and } T_{j} \text { are full days and are considered to be } \\
& \text { exact measurements; therefore they don't affect significant figures.] } \\
& \delta=-23.4^{\circ} \cos \left(\frac{360(9+10)}{365}\right)=-\left(23.4^{\circ} \times \cos (18.73972603)=-\left(23.4^{\circ} \times 0.946987753\right)\right. \\
& =-22.15951342^{\circ}=-22^{\circ}+\left[0.15951342^{\circ} \times\left(\frac{60^{\prime}}{1^{\circ}}\right)\right]=-22^{\circ} \quad 9.57081^{\prime}=-22^{\circ} 9^{\prime}+\left[0.57081^{\prime} \times\left(\frac{60^{\prime \prime}}{1^{\prime}}\right)\right] \\
& \delta=-22^{\circ} 9^{\prime} 34.2483^{\prime \prime}=22^{\circ} 9^{\prime} 34.2^{\prime \prime} \text { South } \leftarrow \text { expected answer in degrees, minutes, seconds }
\end{aligned}
$$

For Weds Jan. $\mathbf{1 0}^{\text {th }} \mathbf{2 0 2 4}$ the latitude where the Sun is directly overhead is:
$T_{J}=10$

$$
\begin{aligned}
\delta & =-23.4^{\circ} \cos \left(\frac{360(10+10)}{365}\right)=-\left(23.4^{\circ} \times \cos (19.7260274)=-\left(23.4^{\circ} \times 0.941317317\right)\right. \\
& =-22.02682523^{\circ}=-22^{\circ}+\left[0.02682523^{\circ} \times\left(\frac{60^{\prime}}{1^{\circ}}\right)\right]=-22^{\circ} 1.609513788^{\prime}=-22^{\circ} 1^{\prime}+\left[0.609513788^{\prime} \times\left(\frac{60^{\prime \prime}}{1^{\prime}}\right)\right] \\
\delta & =-22^{\circ} \\
1^{\prime} 36.5708=22^{\circ} & 1^{\prime} \quad 36.6^{\prime \prime} \text { South } \leftarrow \text { expected answer in degrees, minutes, seconds }
\end{aligned}
$$

Latitudes in the southern hemisphere are represented by a negative (-) value, while latitudes in the northern hemisphere are represented by a positive (+) value.

The Sun being directly overhead at these latitudes makes sense. At this time of year, we are between the Winter Solstice ( $\sim$ Dec. 22 when the Sun is directly over $-23.4^{\circ}$ latitude or $23.4^{\circ}$ South) and the Spring Equinox ( $\sim$ March 21 when the Sun is directly over $0^{\circ}$ latitude or the Equator).

The longitude on Earth where the Sun is directly overhead depends on the time of day. To determine the longitude on Earth where the sun is overhead at the start of your lab you must understand the hour angle, how Earth spins from East to West, that at your solar noon the sun is directly over your longitude and the diagrams below.

Earth spins one revolution every day. So, Earth travels 360 degrees in 24 hours ( $360^{\circ} \div 24$ hours $=$ $15^{\circ}$ per hour). The hour angle ( $\boldsymbol{h}$ ) is used to indicate the longitude where the Sun is directly overhead by adding or subtracting it to your longitude.

You also need to understand the latitude and longitude coordinate system, especially that the world's longitude lines are counted as two $180^{\circ}$ hemispheres (Western hemisphere, Eastern hemisphere) rather than one $360^{\circ}$ sphere. Zero degrees $\left(0^{\circ}\right)$ longitude is at Greenwich (a suburb of London, England). Longitude lines are counted in both the east and west directions, until you reach $180^{\circ}$ at the International

Date Line. This is why it is important to record longitude as degrees West or East. The following pictures can help visualize this.


A 3-dimensional view of Earth, showing longitude (blue) and latitude lines (red).
Latitude is reported as some number of degrees between 0 and 90 , north or south of the equator.
Longitude is reported as some number of degrees between $0^{\circ}$ and $180^{\circ}$. Zero degrees longitude is in Greenwich, (near London) England; and $180^{\circ}$ is the International Date Line. Longitude lines increment from $0^{\circ}$ to $180^{\circ}$ in both east and west directions.

Earth's longitude and latitude lines and how they

## For classes starting at 8:30 am:

At 8:30 PST, in Prince George, the Sun is $\left(-3.5 \mathrm{hr} \times 15^{\circ} / \mathrm{hr}\right)=\left(45^{\circ}+7.5^{\circ}\right)=-52.5^{\circ}$. This means the sun is $52.5^{\circ}$ of longitude East of Prince George (as $8: 30 \mathrm{am}$ is before solar noon). So, the longitude where the Sun is directly overhead at 8:30 am Prince George time is:
$122^{\circ} 41^{\prime} \mathrm{W}-52^{\circ} 30^{\prime}=70^{\circ} 11^{\prime} \mathrm{W}$
Explanation: We have assumed solar noon is the same as local noon. At noon, the longitude where the Sun is directly overhead is the same as Prince George's longitude ( $122^{\circ} 41^{\prime} \mathrm{W}$ ). Since the world spins West to East (counter clockwise when viewed from the North Pole), the longitude where the Sun is directly overhead at the start of our lab is before Prince George's longitude (at a smaller longitude as longitude lines decrease to $0^{0} \mathrm{~W}$ at Greenwich, England). Thus, we subtract to determine the answer.

## For classes starting at 11:30 am:

At 11:30 PST, in Prince George, the Sun is $\left(-0.5 \mathrm{hr} \times 15^{\circ} / \mathrm{hr}\right)=-7.5^{\circ}$ of longitude before solar noon. So, the longitude where the Sun is directly overhead at 11:30 am Prince George time is:
$122^{\circ} 41^{\prime} \mathrm{W}-7.5^{\circ}=122^{\circ} 41^{\prime}-7^{0} 30^{\prime}=115^{\circ} 11^{\prime} \mathrm{W}$
Explanation: Same as above.

For classes started at 2:30 p.m. (or 14:30 PST):
At 14:30 PST, in Prince George the Sun is $\left(2.5 \mathrm{hr} x 15^{\circ} / \mathrm{hr}\right)=37.5^{\circ}$ of longitude past solar noon. So, the longitude where the Sun is directly overhead at 14:30 (2:30 pm) Prince George time is:
$122^{\circ} 41^{\prime} \mathrm{W}+37.5^{\circ}=122^{\circ} 41^{\prime}+37^{0} 30^{\prime}=160^{\circ} 11^{\prime} \mathrm{W}$
Explanation: Now we must add the degrees traveled since noon as the longitude where the Sun is overhead is to the west of Prince George (or past Prince George's longitude).

