1 Lecture Notes

The information in this section (and in Lab Manual Appendix C) is supplementary to the lecture part of the course, and contains material from **a few of** the lectures that is not contained in the textbook. It is included here to free students from having to meticulously copy formulae, and should be brought to lectures when it is being discussed in class. Half of the page is left blank, so that students can add their own notes or diagrams from reading and/or lecture material.

1.1 Energy and its Dimensions and Units

Energy is "the ability to do work". We can express energy in units of work, and there are several systems of units for doing this (see Lab Manual, Appendix D). We use the S.I. system in this course. There are 4 base dimensions from which all other quantities can be derived: length (L), mass (M), time (T) and temperature (θ). The important derived quantities are (dimensions are in parentheses and the corresponding S.I. units in square brackets):

- **Force** is the "push" required to accelerate unit mass (M) [kg] at a unit of distance per unit time per unit time (M L T^{-2}) [kg m s^{-2} = 1N (Newton)].

- **Work** or **Energy** is a unit of force (M L T^{-2}) [kg m s^{-2} = 1N (Newton)] displaced through a unit of distance: (M L^2 T^{-2} = Q) [kg m^2 s^{-2} = N m = 1 J (Joule)].

- **Power** or **Heat flux** is the energy flow per time or the rate of energy flow (rate of work) (M L^2 T^{-3} = Q T^{-1}) [J s^{-1} = 1 W (Watt)].

- **Heat Flux Density** is rate of energy gained or lost unit surface area (M T^{-3} = Q L^{-2} T^{-1}) [W m^{-2}].

1.2 Forms of Atmospheric Energy

Energy can be found in many forms, the 5 of greatest significance in climatic applications are:

1. RADIANT ENERGY: energy associated with electromagnetic waves, requires no medium (i.e. can travel through space)

2. KINETIC ENERGY: energy due to motion:

$$\mathrm{KE} = \frac{mV^2}{2}$$

   m = mass (M), V = speed (LT^{-1}) $\longrightarrow$ so, (KE) = (ML^2 T^{-2} = Q)

3. GEOPOTENTIAL ENERGY: energy derived from gravity:

$$\mathrm{GPE} = mgh$$

   m = mass (M), g = gravitational acceleration (L T^{-2}), h = height (L) $\longrightarrow$ (GPE) = (M L T^{-2} L) = M L^2 T^{-2} = Q)

4. INTERNAL ENERGY: sensible heat (heat that can be felt) of a body due to random motion of molecules:

$$\mathrm{IE} = mcT$$

   m = mass (M), c = specific heat (Q M^{-1} θ^{-1}) T = temperature (θ) $\longrightarrow$ (IE) = (M Q M^{-1} $\theta^{-1}\theta$ = Q)

5. LATENT HEAT: energy involved in phase changes of a substance, especially water:

$$\mathrm{LE} = mL$$

   m = mass (M), L = latent heat (Q M^{-1}) $\longrightarrow$ (LE) = (M Q M^{-1} = Q)

1.3 Conservation of Energy (1st Law of Thermodynamics)

Energy can be neither created nor destroyed, but it can be converted from one form to another (i.e. between forms 1-5 above).

Example: Solar radiation (form 1), heats surface (4), warms air which rises (2 and 3) and evaporates water (5).

1.4 Energy Transfer Processes

1. **Radiation** — heat transfer due to rapid oscillations of electromagnetic fields. May also be considered as waves.

2. **Conduction** — heat transfer by internal molecular activity within a substance with no net external motion. Requires contact between molecules in a substance. Solids, especially metals, are good conductors of heat while liquids and gases are poor due to lower molecular density. In atmosphere conduction is negligible except within the first few millimeters from a surface.

3. **Convection** — heat (and mass) transfer within a fluid by mass motion resulting in transport and mixing. Convection is very important in the atmosphere. Two types of convective motion exist:

   - Free – buoyancy due to thermal differences (war air is less dense, so will rise; cold air is more dense and will sink).
   - Forced – due to physical overturning via shear (air flowing over a rough surface induces vertical motion).

1.5 Radiation Laws

Conservation of Radiant Energy

Radiant energy incident on a body may be reflected, transmitted or absorbed, so that:

$$r_\lambda + t_\lambda + a_\lambda = 1$$

where r_λ = reflectivity (the fraction of incident radiation that is reflected), t_λ = transmissivity (the fraction transmitted), a_λ = absorptivity (the fraction absorbed), of the body for radiation of wave-length λ. This can also be applied to bands of many wavelengths. For the band of solar radiation wavelengths, r_λ is called the albedo (α).

Planck's Law

Relates the way in which the "emissive power" (total energy emitted by a body) of a black body (a perfect emitter) is dependent upon its temperature at all wavelengths. This is the law from which Wien's Law and the Stephan-Boltzmann Law are derived.

Wien's Law

States that a rise in temperature of a body increases its emissive power and also increases the proportion of shorter wave lengths which it emits:

$$\lambda_{\text{max}} = \frac{2.88 \times 10^{-3}}{T_0}$$

where λ_{max} = wavelength of maximum emission (m) T_0 = surface temperature (K)

Stefan-Boltzmann's Law

States that the total energy emitted by a black body, integrated over all wavelengths, is proportional to the fourth power of its absolute temperature (Temperature in Kelvins):

$$I = \sigma T_0^4$$

where I = energy emitted by the black body, σ = Stefan-Boltzmann constant $= 5.67 \times 10^{-8}$W m^{-2} K^{-4}

For non-blackbodies inclusion of the surface emissivity (ϵ_0) allows calculation of the emission:

$$E = \epsilon_0 \sigma T_0^4$$

Kirchhoff's Law

Assuming no transmission through a body, it follows that for a given wavelength and temperature the absorptivity of a body equals its emissivity:

$$a_\lambda = \epsilon_\lambda$$

1.6 Shortwave Radiation (0.15 - 3 μm)

Extra-Terrestrial Distribution

The *solar constant* (I_0) is the amount of solar radiation received from outside the Atmosphere in one unit time on one unit surface area placed perpendicular to the solar beam (as in plane CD in the diagram below) at the Earth's mean distance from the Sun. Present estimates suggest a value of 1367 W m^{-2} which therefore represents the upper limit for solar radiation receipt in the Earth-Atmosphere system. Only locations where the Sun can be directly overhead can receive this value (e.g. in the Tropics). The extra-terrestrial solar radiation (I) received at all other latitudes is less than I_0 and is given by the cosine law of illumination. For example I on the plane AB (representing the top of the Atmosphere) in the following figure is given:

$$I = I_0 \cos Z$$

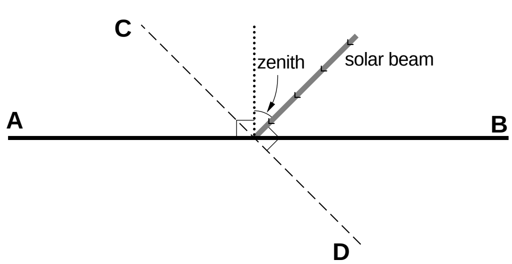

Figure 1: Radiation incident on a surface.

where I_0 = solar radiation received on plane CD perpendicular to beam (i.e. conforms to definition of solar constant), Z = angle between the beam and a perpendicular to the surface (zenith angle);

The zenith angle depends on latitude, season and time of day and gives rise to considerable variation in the amount of energy received over the exterior of the planet.

Atmospheric Attenuation

Further variations in solar radiation receipt at the surface are due to differing path lengths of radiation through the atmosphere due to Earth-Sun geometry and the effectiveness of atmospheric attenuation.

Attenuation Processes

- **Absorption:** the atmosphere is a relatively poor and selective absorber of shortwave. The principal agents are O_3, cloud droplets, particles and water vapour.
- **Scattering:** small gaseous molecules scatter or diffuse shortwave radiation. The shortest wavelengths are preferentially scattered.
- **Reflection:** like a mirror, solar radiation is reflected from larger particles and dominantly by clouds. The reflectivity, or albedo (α, ranges between 0 and 1 with a maximum value of 1 for a perfect reflector like a mirror) of cloud tops is between 0.4 and 0.8 with a mean of 0.55.

Solar radiation reaching the surface ($K\downarrow$) has 2 components: direct-beam (S) as a parallel stream from the solar disc, and diffuse (D) from all points of the sky hemisphere (having been scattered and reflected during passage through the Atmosphere) so that:

$$K\downarrow = S + D$$

Surface Shortwave Radiation Budget

For opaque surfaces (transmissivity $t_\lambda = 0$) shortwave radiation is either reflected or absorbed. Reflection ($K\uparrow$) depends on the amount of incident radiation ($K\downarrow$) and the surface albedo (α):

$$K\uparrow = \alpha K\downarrow$$

The absorbed (or net) shortwave radiation at the surface (K^*) is therefore:

$$K^* = K\downarrow - K\uparrow = K\downarrow (1 - \alpha)$$

1.7 Longwave Radiation (3 - 100 μm)

All bodies with temperature above absolute zero radiate energy consistent with their surface temperature and surface emissivity as given by Stefan-Boltzmann's Law:

$$E = \epsilon_0 \sigma T_0^4$$

At temperatures typical in the Earth-Atmosphere system the wave-length of emission of a body corresponds to infra-red or longwave radiation. The Atmosphere is a relatively good (but still selective) absorber of this longwave radiation. Important absorbers are: water vapour (5-8 μm and at $> 15\mu$m); CO_2 (13-17 μm and 4.3 μm); O_3 (9.4-9.8 μm and 15 μm); and cloud droplets / particles at almost all longwave wavelengths.

There is a significant gap between 8 and 13 μm - called the "atmospheric window". Overall the atmosphere is a relatively good absorber of longwave radiation allowing comfortable living conditions to exist on Earth, hence the so-called *Greenhouse* (or better *Atmosphere*) *Effect*. The Atmosphere largely allows shortwave in but effectively traps a lot of longwave emitted from the surface. This results in a warming of the atmosphere which increases the $L\downarrow$ (back radiation) from the atmosphere to the Earth's surface. The back radiation effectively warms the average surface temperature by 33 K over what it would be if the atmosphere did not exist.

Kirchhoff's law tells us that a good absorber is a good radiator at the same wavelength. The Atmosphere radiates longwave, some to space and some to the ground ($L\downarrow$). The surface also radiates longwave to the atmosphere ($L\uparrow$).

The surface longwave radiation budget therefore consists of two components:

$$L^* = L\downarrow - L\uparrow$$

if the surface is a black body (ϵ = 1):

$$L^* = L\downarrow - \sigma T_0^4$$

but if the surface is a 'grey body' with $\epsilon < 1.0$ we need to allow for less emission but some reflection (reflection = $(1 - \epsilon_0)L\downarrow$):

$$L^* = L\downarrow - (\epsilon_0 \sigma T_0^4 + (1 - \epsilon_0)L\downarrow)$$

Usually $L\downarrow < L\uparrow$ therefore L^* is negative. With clear skies L^* is typically about -100 W m^{-2} for surfaces with an unobstructed view of the sky. The addition of cloud and/or horizon obstructions (e.g. trees, leaves, buildings, topography) substantially lessens or even reverses the radiation losses by increasing $L\downarrow$.

1.8 All-Wave Radiation Budget

The net all-wave radiative budget of a surface (i.e. whether it is gaining or losing energy over all wavelengths) is the net result of the short- and longwave budgets, so that during the day:

$$\begin{aligned} Q^* &= K^* + L^* \\ &= K\!\downarrow - K\!\uparrow + L\!\downarrow - L\!\uparrow \end{aligned}$$

At night $Q^* = L^*$ as $K^* = 0$. Normally by day the radiative budget of a surface is in surplus (Q^* is positive) and by night in deficit (Q^* is negative).

1.9 Surface Energy Balance

Since a surface is a "massless plane" (so thin that it has no mass) it can have no heat content and therefore the Law of Conservation of Energy requires that the radiative energy imbalances be dissipated. This is accomplished via convection or conductive exchanges towards or away from the surface:

$$Q^* = Q_H + Q_E + Q_G$$

where Q_H, Q_E are the convective transfers of sensible and latent heat to or from the atmosphere respectively and Q_G is the conductive transfer of sensible heat to or from the ground. Each of these fluxes may be an energy gain (when directed towards) or a loss (directed away) for the surface. The sign convention for Q_H, Q_E, Q_G is that positive values represent energy flows **away from** the surface, and negative values represent energy flows **toward** the surface. Thus a surplus of radiant energy at the surface (positive Q^*) results in the flow of sensible or latent heat away from the surface (positive Q_H, Q_E and/or Q_G). In moist environments the daytime radiative energy surplus is primarily dissipated as latent heat through the evaporation of water from the surface (Q_E).

1.10 Water Balance

Hydrologic Cycle

A continuous transfer of water occurs through transport and phase changes between subsystems of the Earth-Atmosphere system. Water is evapotranspired (evaporated and transpired through plant respiration) into the atmosphere in response to the local energy balance. Uplift leads to cooling, condensation and precipitation (rain, snow, etc.) thus returning the water to the surface again. There are also transports of water within the atmosphere by advection, across land by river run-off, and through the ground as ground water.

Water Balance at a Site

Surface plane	$p = E + f + \Delta r$
Soil column	$p = E + \Delta r + \Delta S$

where, p = precipitation, E = evapotranspiration, f = infiltration, Δr - net runoff and ΔS net soil moisture storage. Usually p is the sole input, the other terms are outputs or storage terms, but it is possible for E (as dewfall), Δr, or irrigation to be inputs.

The Energy and Water Balances are linked by the energy required to change the phase of water:

$$Q_E = L_V E$$

where, L_V = latent heat of vaporization.

Since p is not a continuous input but occurs as an on/off process Water Budgets normally refer to periods of few days or longer.

1.11 Elementary Thermodynamics of Gases

Our understanding of convective transfer is greatly aided by knowledge of 3 simple thermodynamic laws: *Ideal Gas Law; Hydrostatic Law; 1st Law of Thermodynamics*

1. **Ideal Gas Law (Equation of State for the Atmosphere)**
   Gases consist of minute molecules in a state of irregular motion. The pressure (P) of a gas results from the impacts of these molecules and for one unit volume of gas, depends on:

   (a) number of molecules

   (b) mass of molecules

   (c) speed of molecules

   a) and b) define the density (ρ) of the gas (ie. $no. \times \frac{mass}{volume}$), and c) depends upon the gases temperature (T). These are related by:

   $$P \propto \rho T$$

   or

   $$P = R\rho T$$

   where R is the Specific Gas Constant ($R = \frac{R^*}{M_{air}}$ where R^* is the universal gas constant = 8.314 kg m^2 s^{-2} mole^{-1} K^{-1}; and M$_{air}$ is the molecular weight of "air" = .028 kg mole^{-1} (This is the same as the more familiar form: PV = nR*T, except that n, the number of moles of gas is replaced by $\frac{m}{M_{air}}$ where m is the mass of gas under consideration.)

2. **Hydrostatic Law**
Consider an air column divided into thin horizontal slices of thickness Δz with cross sectional area a. Then the *volume* of a slice is $a\Delta z$. If ρ is air density, the *mass* of the slice is $\rho a\Delta z$. If the acceleration due to gravity is g, the *force* on the slice (F=ma) is $g\rho a\Delta z$ and the *pressure* (pressure is force per unit area) at A due to the slice is $\frac{g\rho a\Delta z}{a} = g\rho\Delta z$. Thus the pressure difference ΔP decreases with height:

$$-\Delta P = g\rho\Delta z$$

where the sign indicates that P decreases as z increases. This is the Hydrostatic Equation, and is also a statement of hydrostatic balance: the vertical decrease of pressure tending to cause uplift is balanced by the downward weight of the air. Hence there is no vertical acceleration and the atmosphere does not float away (luckily!).

3. **1st Law of Thermodynamics**
This is a Law of Conservation of Energy, and a statement of the physical changes resulting when heat is added to, or taken away from a gas. For solids, recall that there is a direct relationship between the heat added (ΔQ) and the corresponding temperature change (ΔT):

$$\begin{aligned} \Delta Q &\propto \Delta T \\ \Delta Q &= c\Delta T \end{aligned}$$

With a gas we have to consider whether the gas is capable of expansion. If not, (ie. V is constant) addition of heat will result in a greater value of ΔT than if it is able to expand because in the latter case some of the energy is used to do the work of expansion. So for gases we have two specific heats: one for constant volume (C_V), and the other for constant pressure (C_P), and the 1st Law of Thermodynamics is:

$$\begin{array}{cccccc} \Delta Q & = & C_V \Delta T & + & P/(\Delta\rho) \\ \Delta Q & = & C_P \Delta T & - & (\Delta P)/\rho \\ \text{change} & = & \text{change in} & \pm & \text{work due to} \\ \text{in heat} & & \text{internal energy} & & \text{expansion} \end{array}$$

The second form, involving C_P is most useful, because the changes ΔT, and ΔP are easy to measure, whereas $\Delta\rho$ is not. The value of C_P for air is 1010 J kg^{-1} K^{-1}. C_P, C_V, and R (the gas constant for air) are related by:

$$\begin{array}{ccccc} R & = & C_P & - & C_V \\ 287 & = & 1010 & - & 723\ [\text{J kg}^{-1}\ \text{K}^{-1}] \end{array}$$

1.12 Forces in the Atmosphere

Encapsulating Questions

1. What primary laws of physics does atmospheric motion obey?
2. What are meant by *real* and *apparent* forces?
3. What *real* forces are important in the atmosphere? What is their affect?
4. What *apparent* forces are important in the atmosphere? What is their affect?
5. How does wind (horizontal motion) result from a balance of the forces acting on an air parcel?
6. Geostrophic, Gradient, and Cyclostrophic winds are approximations to the full equation of motion. When are each of these valid, and what are the assumptions upon which these models are based?

Atmospheric Motion

Motion in the atmosphere behaves according to Newton's First Law:

> *In the absence of forces, a body in motion will remain in motion.*

but it is Newton's second law that provides the basis for the equations of motion:

$$\vec{\mathbf{F}} = m\vec{\mathbf{a}}$$

which can be re-expressed as:

$$\vec{\mathbf{a}} = \frac{d\vec{\mathbf{V}}}{dt}(\simeq \frac{\Delta\vec{\mathbf{V}}}{\Delta t}) = \sum \frac{\vec{\mathbf{F}}}{m}$$

where $\vec{\mathbf{V}}$ = velocity, $\vec{\mathbf{a}}$ = acceleration, $\vec{\mathbf{F}}$ = force, and m = mass. So, what are the important forces in the atmosphere?

Real forces:

- The horizontal Pressure Gradient Force ($\vec{\mathbf{PGF}}$) acts from high toward low pressure and is proportional to the change in pressure divided by the distance over which the change occurs: $\vec{\mathbf{PGF}}$ = $-\frac{1}{\rho}\frac{\Delta P}{\Delta x}$. One can also express the horizontal pressure gradient force as proportional to the horizontal change in height of a constant pressure surface ($\frac{\Delta Z_p}{\Delta x}$) which is related to the first representation through the hydrostatic equation $\Delta P = -\rho g \Delta Z$. If the hydrostatic equation is used to replace ΔP with ΔZ_p, then $\vec{\mathbf{PGF}}$ = $g\frac{\Delta Z_p}{\Delta x}$. [NOTE: The textbook uses the symbol F_p to represent $\vec{\mathbf{PGF}}$.]

- Friction ($\vec{\mathbf{F_f}}$) acts in opposition to the velocity.

Apparent forces are due to the earth's rotation. The earth is a non-inertial frame of reference — the frame of reference itself is accelerating because of the rotation. Because of this, we must add these *apparent* forces to the equation of motion on a sphere:

- Coriolis Force ($\vec{\mathbf{C}}$). The Coriolis force varies with latitude, is proportional to and acts to the right of the velocity (in the northern hemisphere). It is expressed as fV, where f is called the Coriolis parameter and is equal to 2 times the rotation rate of the earth times the sine of the latitude. [NOTE: The textbook uses the symbol F_C to represent $\vec{\mathbf{C}}$.]

- Centrifugal Force. The centrifugal force is included in gravity.

The force balance in the atmosphere relates the acceleration of an air parcel to the sum of forces acting on it:

$$\frac{\Delta \vec{\mathbf{V}}}{\Delta t} = \mathbf{P}\vec{\mathbf{G}}\mathbf{F} + \vec{\mathbf{C}} + \vec{\mathbf{F_f}}$$

If we use a *natural coordinate system,* where s is the coordinate along the direction of motion, n is the coordinate to the left and perpendicular to the direction of motion, $\vec{\mathbf{s}}$ points in the direction of s, and $\vec{\mathbf{n}}$ points in the direction of n.

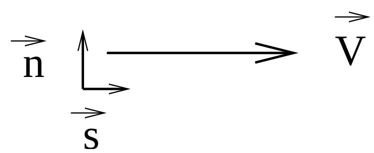

Figure 2: Natural coordinate system. $\vec{V}$ represents the wind vector. $\vec{s}$ is the unit vector pointing in the direction of motion. $\vec{n}$ is the unit vector pointing perpendicular to the direction of motion.

$$\vec{\mathbf{V}} = V\vec{\mathbf{s}}$$

$$V = \frac{\Delta s}{\Delta t}$$

and

$$\frac{\Delta V\vec{\mathbf{s}}}{\Delta t} = \frac{\Delta V}{\Delta t}\vec{\mathbf{s}} + V\frac{\Delta \vec{\mathbf{s}}}{\Delta t}$$

where

$$\vec{\mathbf{C_f}} = V\frac{\Delta \vec{\mathbf{s}}}{\Delta t}$$

is a centrifugal force for curved flow, and is equal to

$$\frac{V^2}{R}\vec{\mathbf{n}}$$

where R is the radius of curvature and is > 0 for curvature to the left and < 0 for curvature to the right. So we have, in natural coordinates:

$$\vec{\mathbf{a}} = \frac{\Delta \vec{\mathbf{V}}}{\Delta t} = \frac{\Delta V}{\Delta t}\vec{\mathbf{s}} + \frac{V^2}{R}\vec{\mathbf{n}}$$

How do the forces fit into the natural coordinate system?

The Pressure Gradient Force $\vec{\mathbf{PGF}}$ can have components along and across the direction of motion:

$$\vec{\mathbf{PGF}} = -\frac{1}{\rho}\frac{\Delta P}{\Delta s}\vec{\mathbf{s}} + -\frac{1}{\rho}\frac{\Delta P}{\Delta n}\vec{\mathbf{n}}$$

The Friction Force $\vec{\mathbf{F_f}}$ always acts in a direction opposite to the velocity:

$$\vec{\mathbf{F_f}} = -F_f\vec{\mathbf{s}}$$

The Coriolis Force $\vec{\mathbf{C}}$ always acts to the right of the motion (in the northern hemisphere):

$$\vec{\mathbf{C}} = -fV\vec{\mathbf{n}}$$

So, if we add all of these forces up, and write them as two equations, one with force components in the direction of motion $\vec{\mathbf{s}}$, and one with force components perpendicular to the direction of motion $\vec{\mathbf{n}}$ we get, the force balance:

$$\frac{\Delta\vec{\mathbf{V}}}{\Delta t} = \vec{\mathbf{PGF}} + \vec{\mathbf{C}} + \vec{\mathbf{F_f}}$$

which in the $\vec{\mathbf{s}}$ direction (along the flow) gives:

$$\frac{\Delta V}{\Delta T} = -\frac{1}{\rho}\frac{\Delta P}{\Delta s} - F_f \tag{1}$$

and in the $\vec{\mathbf{n}}$ direction (across the flow):

$$\frac{V^2}{R} = -fV - \frac{1}{\rho}\frac{\Delta P}{\Delta n} \tag{2}$$

1.13 Application of the Equation of Motion in Natural Coordinates

Imagine an initial state with a horizontal pressure gradient and no motion. Equation 2 implies that $\frac{\Delta P}{\Delta n} = 0$ therefore, the initial flow will be down the pressure gradient (across the isobars). As time proceeds, V becomes non-zero, and the wind accelerates until a three way balance exists between $\vec{\mathbf{PGF}}$, $\vec{\mathbf{C}}$, and $\vec{\mathbf{F_f}}$.

Geostrophic Wind

Assume that the flow is straight (ie. $R \implies \infty$), steady (ie. $\frac{\Delta V}{\Delta t} = 0$), and frictionless ($F_f = 0$). These conditions will generally be met to within a few percent above a couple of kilometers, and in regions where the isobars are straight. Equation 1 implies that $\frac{\Delta P}{\Delta s} = 0$ which means that the flow is parallel to the isobars. Equation 2 implies that the Pressure gradient force is balanced by the Coriolis force:

$$fV = -\frac{1}{\rho}\frac{\Delta P}{\Delta n}$$

which is the **Geostrophic Wind** relationship. An alternate expression can be found for the geostrophic wind by using the height gradient form of the pressure gradient force:

$$fV = g\frac{\Delta Z_p}{\Delta n}$$

Gradient Wind

The gradient wind approximation relaxes the requirement for straight flow. Again, equation 1 implies flow parallel to the isobars. For anti-clockwise curvature ($R > 0$) (cyclonic) equation 2 is:

$$fV + \frac{V^2}{R} = -\frac{1}{\rho}\frac{\Delta P}{\Delta n}$$

and for clockwise curvature ($R < 0$) (anticyclonic) equation 2 becomes:

$$fV - \frac{V^2}{R} = -\frac{1}{\rho}\frac{\Delta P}{\Delta n}$$

So, for the same pressure gradient, anticyclonically curved flow will have stronger winds than straight flow, which will have stronger winds that cyclonically curved flow. However in practice, pressure gradients are weaker around anticyclones.

Cyclostrophic Wind

Imagine, a curved circulation which exists on small time and length scales, ignoring friction. In this case, we can ignore the Coriolis force. The pressure gradient force is balanced by the centrifugal force. The flow can be either cyclonic or anticyclonic.

$$\pm\frac{V^2}{R} = -\frac{1}{\rho}\frac{\Delta P}{\Delta n}$$

This equation describes the circulation in a tornado.